

An aerial photograph of the University of Exeter campus, showing green lawns, trees, and buildings, partially framed by a blue curved graphic element.

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Strategic Advance Production

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Abstract : Advance production serves as a means of quantity commitment. Therefore an oligopolist, unlike a monopolist, may have an incentive to invest in advance production in order to pre-empt its opponent(s) even when [i] it is technologically more costly than on-spot production, and [ii] it does not entitle the firm to Stackelberg leadership in the subsequent marketing stage. When firms set quantities, such pre-emption acts as strategic substitutes between oligopolists. Namely, in a pure strategy subgame perfect equilibrium, *some* but *not all* firms may engage in advance production, whether the firms are *a priori* symmetric or not. More generally, a firm's incentive for advance production arises only if there is a quantity-setting opponent, irrespective of the firm's own strategic variable (i.e., price or quantity) and the characteristics of the concerned products (i.e., substitutes or complements).

Keywords : inventory, storage costs, time preferences, pre-emption, strategic substitution.

JEL classification : L13, D43, E22.

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1 Introduction

INVESTMENT IN CAPACITY is usually modelled under the assumption that advance investment saves total production costs, or alternatively put, additional production beyond the planned capacity is cost-inefficient. A typical total cost function is

$$TC[q|k] = \begin{cases} \gamma k + wq & \text{if } q \leq k, \\ (\gamma + w)k + (\gamma + \hat{w})(q - k) & \text{if } q > k, \end{cases}$$

where q is the production quantity, k is the planned capacity, γ is the unit cost of capacity, and the marginal production costs within and beyond the planned capacity are w and \hat{w} , respectively, where $w < \hat{w}$.¹ In this type of situation, a firm may choose to invest in capacity *even when the firm is a monopolist facing no competition*. Literature on oligopoly games with capacity investment has been pioneered by Levitan and Shubik (1972) followed by Kreps and Scheinkman (1983), Davidson and Deneckere (1986), *inter alia*. The main spirit of this literature is to regard capacity as a pseudo-quantity variable *while the main (short-term) strategic variable is the price*. Hence, this literature can be viewed as a theoretical attempt to rectify obvious flaws which used to inhere in traditional Bertrand oligopoly theory.

This well-established literature leaves two spontaneous questions unaddressed. First, when firms have incentives for precommitted capacity investment, is it due to strategic effects or purely non-strategic cost efficiency in production? Second, is there any role which capacity, or some other forms of production commitment, can play in non-Bertrand oligopoly? In other words, should capacity investment necessarily and exclusively be a replacement for a quantity variable?

To address the first question, in this paper we present an alternative model of production commitment, in which advance investment brings no cost reduction at all. More precisely, we model a kind of commitment in which advance investment is strictly more expensive than instantaneous production. In the real world, advance production as inventory investment gives an example of this model, because of the costs associated

¹In Kreps and Scheinkman (1983), for instance, $\gamma > 0$, $w = 0$ and $\hat{w} = \infty$. In the context of a sequential entry game, Dixit (1979, 1980) showed how an incumbent gains a strategic advantage by investing in capacity prior to the entry of a competitor, using a model in which $\gamma > 0$, $w = \hat{w}$.

with storage. In addition, unless there is technological advantage made possible *only* by advance investment, it is always less economical to incur investment ahead of time, due to time preferences. Therefore, in the absence of competition, a firm would never make any advance investment.

In the presence of competition, however, advance production can serve as a means of strategic pre-emption. This is precisely how quantity and capacity can coexist as two separate strategic variables with each bearing a distinct strategic mission, relating to our aforementioned second question. Saloner (1987) models two-stage Cournot duopoly where there is neither cost advantage nor disadvantage in advance investment, but advance production entitles the firm to Stackelberg leadership. This is followed by Pal (1991) with a more general setting, taking into account the differential in production costs between the two stages. It is shown that, with the possibility of Stackelberg leadership, firms may have advance production incentives even if production is more (but not too much more) expensive in the first stage compared to the second stage.

As is well known, Stackelberg leadership by itself yields an unambiguous profit improvement as opposed to simultaneous-move Nash equilibria. This has been shown in Henderson and Quandt (1971)², Gal-Or (1985)³, and a large number of subsequent contributions. As is well known, this also forms the basis of more recent literature on endogenous timing games, pioneered by Hamilton and Slutsky’s “extended games” (1990)⁴ and developed further by Pal (1996), *inter alia*.

²If Cournot firms were to choose their action timing endogenously, (implicitly assuming no demand uncertainty or informational spillover thereof) all firms would want to be leaders whilst no firm would volunteer to be a follower, hence Stackelberg “equilibria” would never be sustainable. Henderson and Quandt called this “Stackelberg disequilibrium.”

³Leaders tend to be more (resp., less) profitable than followers when actions are strategic substitutes (resp., complements), but in either case, being a leader (i.e., capable of precommitment) is unequivocally more profitable than being a simultaneous Nash-equilibrium player (i.e., incapable of precommitment).

⁴According to Hamilton and Slutsky, extended games have two versions: “extended games with observable delay” and “extended games with action commitment.” The difference between these two versions is that only in the former, not in the latter, can each firm observe the other’s timing decision *before* setting its own marketing action such as a supply quantity. The version directly related to our endogenous timing decisions is the former.

This inevitably implies that, in those preceding studies by Saloner (1987) and Pal (1991), the incentives for advance production are assisted by the extra benefit from Stackelberg leadership in the marketing stage. Our objective in this paper is to inspect advance production incentives when not assisted by the prospect of Stackelberg leadership. To this end, we construct a model where the order of moves in the marketing stage is independent of the timing of production. Namely, firms engage in simultaneous-move oligopoly irrespective of the levels of their precommitted investment. We nevertheless derive a partly similar result, i.e., advance investment may indeed take place in equilibrium. Hence our result reflects unambiguously the strategic benefit genuinely from advance production, not the Stackelberg leadership advantage in marketing.

A basic model of quantity-setting oligopoly and the key intuition are laid out in section 2. In spite of *a priori* symmetry of the oligopoly model, there is a range of cost-parameters over which only *some* of the firms can profitably make advance investment. There cannot be a pure strategy subgame perfect equilibrium in which *all* firms invest ahead of time. We thereby derive an endogenous behavioural asymmetry without relying upon the prospect of Stackelberg leadership. We mainly analyse a linear duopoly example *à la* Singh and Vives (1984) to show concrete equilibrium comparative statics results. Note that there are ranges of parameter values over which no pure strategy subgame perfect equilibria can exist. A brief discussion on macroeconomic implications of strategic inventories is provided in section 3.

A qualitative comparison between price competition and quantity competition is discussed in section 4. The essence of our finding is such that a firm can benefit from advance production *only if there is a quantity-setting opponent*. Quite intriguingly, this result stands intact whether the firm itself sets a quantity or a price, and whether the firms' products are mutually substitutes or (imperfect) complements. In other words, an oligopolist's strategic incentives to invest in inventory is affected directly by its opponent's strategic variable, irrespective of the slopes of reaction curves. The intuition can be obtained graphically better than either verbally or algebraically, thus section 4 provides a succinct graphic analysis. Section 5 concludes the paper.

2 The basic model

2.1 Set-up

Consider a two-stage Cournot duopoly model. In the first stage, two single-product firms simultaneously decide on their advance production levels k_1, k_2 . By the end of the first stage, k_1, k_2 become commonly observable. In the second stage, firms compete in a Cournot market, setting (again simultaneously) their supply quantities q_1, q_2 , each firm facing the linear inverse demand function

$$p_i = a - q_i - bq_j$$

where $a > 0$ and $b \in (-1, 0) \cup (0, 1]$ are known constants. Firm i 's total cost discounted to the beginning of the game is

$$C[q_i|k_i] = \begin{cases} \gamma k_i & \text{if } q_i \leq k_i, \\ \gamma k_i + \delta \gamma (q_i - k_i) & \text{if } q_i > k_i, \end{cases}$$

where $\gamma > 0$ is the unit cost of production, and $\delta \in (0, 1)$ is the discount factor which is multiplied only to the on-spot component of the cost. For simplicity, we assume γ to be the same in both periods, hence advance production is costly only due to time preferences δ .

The key assumption in our model is to accept the fact that *advance production is inherently more expensive than on-spot production*. Hereby the most economical way for firm i ($i = 1, 2$) to supply any positive q_i is to set $k_i = 0$. Therefore, it is intuitively obvious that $k_1 = k_2 = 0$ will be a pure strategy subgame perfect equilibrium (simply “equilibrium” hereinafter unless otherwise specified) either when γ is substantially large relative to a , or when δ is close to zero.

Our main interest here is to examine the possibilities:

1. that $k_1 = k_2 = 0$ may not always be an equilibrium,
2. that $k_i > k_j = 0$ ($\{i, j\} = \{1, 2\}$) can be an equilibrium, and
3. whether $k_i \geq k_j > 0$ can ever be an equilibrium.

2.2 Equilibria

Firm i 's nett profit discounted to the beginning of the game is therefore

$$Y_i = \begin{cases} \delta(a - q_i - bq_j)q_i - \gamma k_i & \text{if } q_i \leq k_i, \\ \delta[(a - q_i - bq_j)q_i - \gamma(q_i - k_i)] - \gamma k_i & \text{if } q_i > k_i, \end{cases}$$

and hence, the best reply function is

$$R_i[q_j|k_i] = \max \left\{ \frac{a - \gamma - bq_j}{2}, \min \left\{ k_i, \frac{a - bq_j}{2} \right\} \right\}$$

(see Appendix). Solving the game backward, the equilibrium advance production profiles are characterised as follows.

[0] $k_1 = k_2 = 0$ is an equilibrium iff either

$$\begin{aligned} \frac{1}{\delta} \geq \max & \left\{ \frac{b^2}{2(2+b)} \cdot \frac{a}{\gamma} + \frac{b^3}{2(4-b^2)}, \right. \\ & \left. \left(1 - \frac{b}{2} - \frac{\sqrt{2(2-b^2)}}{2+b} \right) \frac{a}{\gamma} + \left(\frac{b}{2} + \frac{\sqrt{2(2-b^2)}}{2+b} \right) \right\} \end{aligned} \quad (1)$$

or

$$\frac{4}{4-b^2} \left(1 - \frac{\gamma}{(2-b)a + b\gamma} \right) \leq \frac{1}{\delta} < \frac{b^2}{2(2+b)} \cdot \frac{a}{\gamma} + \frac{b^3}{2(4-b^2)}. \quad (2)$$

[1] Asymmetric equilibria $k_i = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}$, $k_j = 0$ ($\{i, j\} = \{1, 2\}$) exist iff

$$\begin{aligned} \max & \left\{ \frac{b^2}{2(2+b)} \cdot \frac{a}{\gamma} + \frac{b^3}{2(4-b^2)}, \frac{((2-b)\sqrt{2(2-b^2)} - 4 + 2b + b^2)a + (4-b^2)\gamma}{2(\sqrt{2(2-b^2)} + b)\gamma} \right\} \\ & \leq \delta \leq \left(1 - \frac{b}{2} - \frac{\sqrt{2(2-b^2)}}{2+b} \right) \frac{a}{\gamma} + \left(\frac{b}{2} + \frac{\sqrt{2(2-b^2)}}{2+b} \right) \end{aligned} \quad (3)$$

[2] An asymmetric profile $k_i = \frac{a}{2+b} + \frac{b\gamma}{4-b^2}$, $k_j = 0$ becomes an equilibrium iff

$$\begin{aligned} \max & \left\{ \frac{4}{4-b^2} \left(1 - \frac{1}{2-b} \cdot \frac{\gamma}{a} \right), \frac{2-b}{2} \cdot \frac{a}{\gamma} - \frac{((2-b)a - 2\gamma)\sqrt{2(2-b^2)}}{(4-b^2)\gamma} \right\} \\ & \leq \delta \leq \min \left\{ \frac{b^2}{2(2+b)} \cdot \frac{a}{\gamma} + \frac{b^3}{2(4-b^2)}, \frac{4}{4-b^2} \left(1 - \frac{\gamma}{(2-b)a + b\gamma} \right) \right\}. \end{aligned} \quad (4)$$

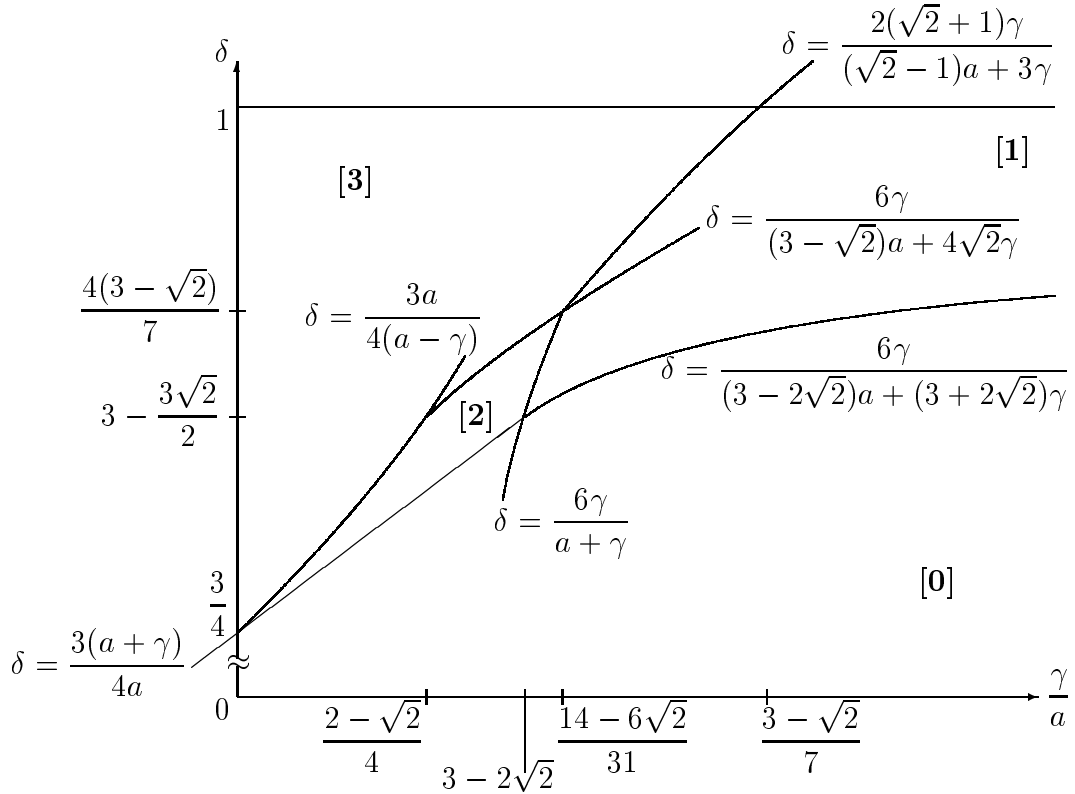
[3] No pure strategy equilibrium exists iff

$$\frac{1}{\delta} < \min \left\{ \max \left\{ \frac{4}{4-b^2} \left(1 - \frac{1}{2-b} \cdot \frac{\gamma}{a} \right), \frac{2-b}{2} \cdot \frac{a}{\gamma} - \frac{((2-b)a - 2\gamma)\sqrt{2(2-b^2)}}{(4-b^2)\gamma} \right\}, \right. \\ \left. \frac{((2-b)\sqrt{2(2-b^2)} - 4 + 2b + b^2) a + (4-b^2)\gamma}{2(\sqrt{2(2-b^2)} + b)\gamma} \right\}. \quad (5)$$

Note that regime [3] becomes empty when $b \in (-1, 0)$.

Detailed algebraic derivation of these equilibria is given in Appendix. For an illustrative purpose, a comparative statics diagram given $b = 1$ (i.e., the two firms are perfectly substitutable suppliers) is given in figure 1.

Figure 1 : equilibrium advance production with $b = 1$.



The implication of these equilibrium results is as follows.

Proposition 1 : There exists a range of parameters $\{a, b, \gamma, \delta\}$ where $k_1 = k_2 = 0$ is an equilibrium.

Proposition 2 : There exists a range of parameters $\{a, b, \gamma, \delta\}$ where the *only* pure strategy subgame perfect equilibria are $k_i > k_j = 0$ ($\{i, j\} = \{1, 2\}$).

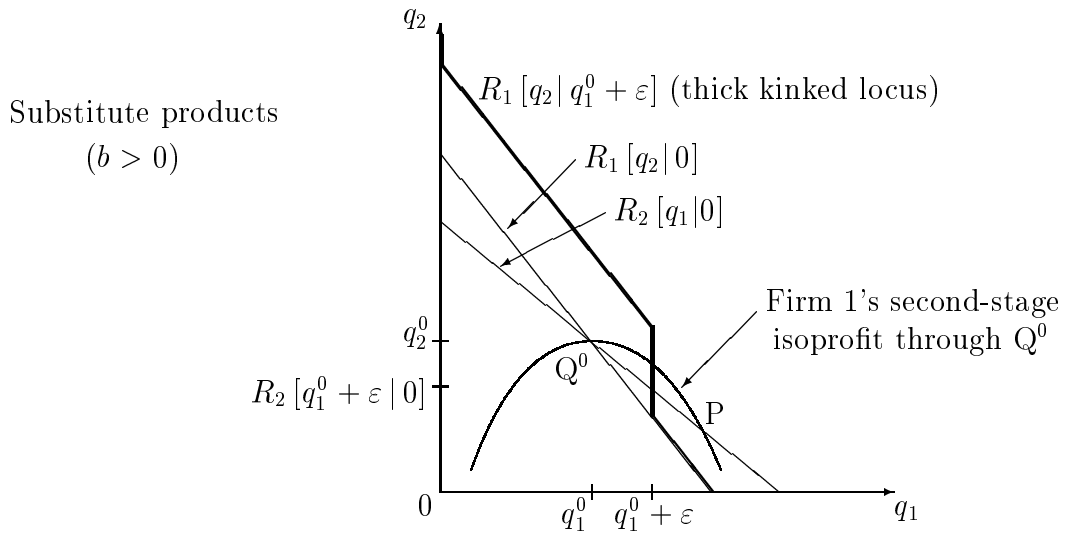
Proposition 3 : When $b \in (0, 1]$, there exists a range of parameters $\{a, \gamma, \delta\}$ where *no* pure strategy subgame perfect equilibrium exists.

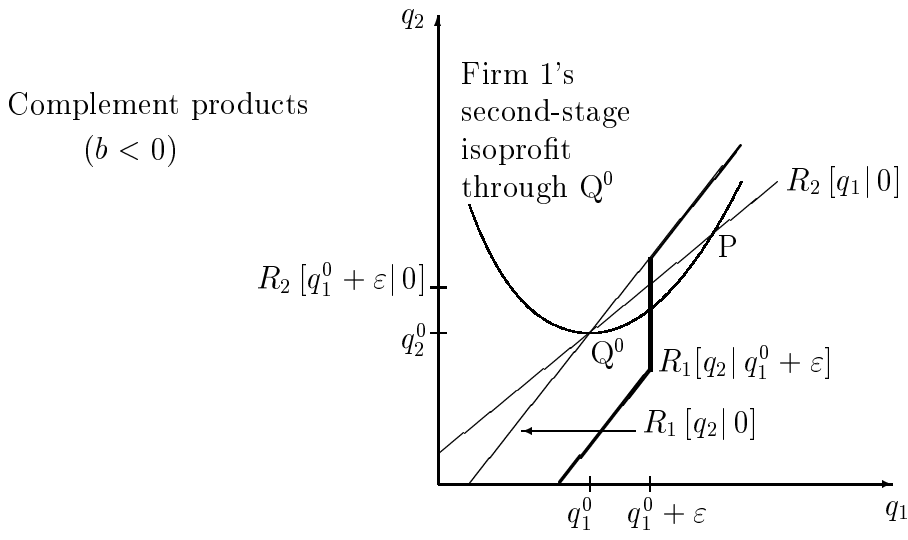
Proposition 4 : For any parameter values a, b, γ and δ , there exists no pure strategy subgame perfect equilibrium such that $k_1 > 0, k_2 > 0$.

Intuition : When δ is low and/or when γ is high relative to a , $k_1 = k_2 = 0$ is an obvious pure strategy equilibrium. Also, when a is low, the market is too thin to make the strategic advantage from advance production attractive enough. This corresponds to case [0], hence Proposition 1.

As δ or a grows higher, one of the duopolists has a strict incentive to produce in advance a strictly positive quantity. A quick intuition can be obtained as follows. With $k_1 = k_2 = 0$ both firms' second-stage reaction curves would be either strictly downward sloping if $b > 0$, or strictly upward sloping if $b < 0$, as illustrated in figure 2. At the same time, in the neighbourhood of the equilibrium quantity profile $Q^0 = (q_1^0, q_2^0)$, firm 1's *isoprofit* would have a zero slope. Thereby firm 1 can always find a segment $\overline{Q^0P}$ of firm 2's reaction function (to the immediate right of Q^0) which lies on the higher-profit side of the isoprofit.

Figure 2 : Firm 1's advance production incentive when $k_2 = 0$.





By advance producing $k_1 = q_1^0 + \varepsilon$, firm 1 can shift its second-stage reaction curve outward, as illustrated by the thickened kinked lines in figure 2. This helps sustain a quantity profile on the segment $\overline{Q^0P}$ which would previously lie on the higher-profit side of the previous equilibrium Q^0 , in exchange for an increase in production costs due to advance production.⁵ This trade-off can resolve in favour of advance production unless δ is excessively low or γ is prohibitively high. This leads to an incentive for one of the firms to engage in advance production, leading to cases [1] and [2], hence Proposition 2.

When a and δ are both high, both firms might have strict incentives for advance production. Thereby $k_1 = k_2 = 0$ cannot be a pure strategy equilibrium. Furthermore, even after one firm has chosen $k_i > 0$, the other firm would still have a strict preference for $k_j > 0$ as opposed to $k_j = 0$. Hence, any pure profile involving $k_j = 0$ cannot be an equilibrium. On the other hand, it can be proven that $k_1 > 0$ and $k_2 > 0$ cannot coexist in equilibrium, hence Proposition 4 (see Appendix for further details). Especially when $b > 0$, this can entail the vacuum of pure strategy equilibria in case [3], hence Proposition 3.

⁵The only relevant “curves” here are the two firms’ reaction curves and firm 1’s *original* isoprofit curve with $k = 0$. When $k_1 > 0$, the change in production costs may cause firm 1’s second-stage isoprofit to shift away from its original position, but this shift is irrelevant to our intuition here. The same applies in figures 3 and 4 (see section 4).

3 Macroeconomic implications

3.1 Stylised facts on hypervolatility of inventories

In almost every economy in the world, consumption is far and away the largest component, comprising more than 80% of the economy's gross production (either GDP or GNP). The remainder is broadly defined investment, or saving. Within investment, inventories are only a tiny fraction, typically less than 1% of the GDP. However, the *fluctuation* of investment tends to be much more substantial than that of consumption. In particular, it has been universally observed that inventory investment is so much more volatile than the remainder of investment that its (detrended) fluctuation can weigh as much as 40 to 50 percent of the total detrended variation of the GDP.⁶ In this section we attempt a quick glance at how our foregoing microtheoretic findings can be potentially utilised to disentangle this classical puzzle in macroeconomics.

The fact that consumption is proportionately smoother than investment, has been explained by the **consumption smoothing** theory.⁷ That is, utility maximisation of each individual consumer entails the tendency of absorbing intertemporal income fluctuations into saving and dissaving, so as to smooth out the consumption level, making investment (which should theoretically equal saving) fluctuant relative to consumption. The difference between inventory and the remainder of investment, however, is less transparent.

An analogue of consumption smoothing theory in an attempt to explain disproportionate volatility of inventory investment is the theory of **production smoothing**. In the presence of capacity and other constraints, it tends to be more economical to keep the level of production constant over time, so that the intertemporal fluctuations in demand, or in sales, tends to be absorbed by inventories. If this is the truth, then it should be expected that the fluctuations in inventories are *countercyclical*. An empirical criticism cast against this theory is the fairly general finding that inventories

⁶For further details, see Blanchard and Fischer (1989), chapter 6.

⁷A pioneering research in this direction can be found in Holt *et al* (1960).

are statistically not countercyclical.⁸

Another theory of inventory investment, which can explain *procyclical* fluctuations, is the **stock out** theory.⁹ This theory is simply based upon the common observation that most commodities *cannot be shortsold*. Hence, whenever the market is booming, suppliers must increase inventories in order to prevent the loss of sales caused by stock-out. However, it remains puzzling why inventories need to be disproportionately more fluctuant than the remainder of the economy, given that all suppliers should rationally expect the overall magnitude of fluctuations in the market, taking into account every consumer's consumption smoothing.

By definition, macroeconomics (except for its “microfoundation” part) does not investigate into *industry specific* inventory behaviour. Hereinbelow we contemplate an alternative explanation to this puzzle, in light of the strategic aspects of oligopolistic industries.

3.2 A hypothetical “industry”

Consider an oligopolistic industry embedded in discrete time where, in every period $t = 1, 2, 3, \dots$, the two-stage duopoly game as in section 2 is played. The demand intercept a varies randomly from a period to another, hence this is not a repeated game in its strict sense. The precise value of a is revealed at the beginning of every period, i.e., before the first stage of the period is played.

Recall once again that advance production is inherently less economical than on-spot production. Hence, in order for any advance production to take place at all, the industry should not be collusive, either tacitly or explicitly. Hereby we assume away any tacitly collusive equilibria, that is, we henceforth assume that firms play a period-by-period equilibrium.

⁸Procyclicality of inventories has been established empirically by Blanchard (1983) and Blinder (1986).

⁹Establishment of the stock out theory is due to Kahn (1987).

3.3 Illustrative numerical example

To develop a quick intuition, apply the following benchmark parametric values to our linear duopoly model in the previous section. Assuming that $b = 1$ and $\gamma \geq 0.23a$ (that is, we focus on the right-hand side area of figure 1 where the possible outcomes are either $[0]$ or $[1]$):

- if $a < \bar{a} = \left(\frac{6(3 + 2\sqrt{2})}{\delta} - (3 + 2\sqrt{2})^2 \right) \gamma$ neither firm invests in advance production and each firm sells $q^{CN} = \frac{a - \gamma}{3}$ at the price $p^{CN} = \frac{a + 2\gamma}{3}$;
- if $a \geq \bar{a} = \left(\frac{6(3 + 2\sqrt{2})}{\delta} - (3 + 2\sqrt{2})^2 \right) \gamma$ one of the firms produces $q^{AP} = \frac{a + \gamma}{2} - \frac{\gamma}{\delta}$ in advance, and sells this quantity (i.e., does not engage in any additional on-spot production) whilst the other firm produces on-spot and sells $q^{OS} = \frac{a - 3\gamma}{4} + \frac{\gamma}{2\delta}$, at the price $p^{AP} = \frac{a + \gamma}{4} + \frac{\gamma}{2\delta}$,

A checkpoint which deserves careful attention is the interpretation of statistical data: although “production” in economic theory is usually defined as the *quantity* of production, in economic statistics it is typically measured by the *money amount* of transaction which, in our duopoly model, equals the *sales revenue* of the firms. When the demand intercept a fluctuates around the aforementioned threshold \bar{a} , it is obvious that the sum of the two firms’ supply *quantity* increases when a crosses the threshold \bar{a} upward and decreases when a falls below, that is, $q^{AP} + q^{OS} > 2q^{CN}$ in the neighbourhood $a \approx \bar{a}$. Nevertheless, in the same neighbourhood $a \approx \bar{a}$, the firms’ total revenues show the relation

$$(q^{AP} + q^{OS})p^{AP} > 2q^{CN}p^{CN} \quad \text{if and only if} \quad \delta < \frac{6 + 5\sqrt{2}}{14} \approx 0.93365.$$

In this sense, if we conceptualise the demand intercept a as a reflexion of macroeconomic performance, the strategic effect of inventories in an oligopolistic industry serves to make the transaction statistics in the industry appear *procyclical* if and only if $\delta > 0.93365$. On the other hand, the amount of inventory investment is always *procyclical*: nil when $a < \bar{a}$ whilst $q^{AP}p^{AP}$ when $a \geq \bar{a}$.

To develop a concrete idea, we plug some numbers in and see how our “strategic explanation” to the extreme volatility of inventory investment appears. Noting on one

hand that our “discount factor” δ incorporates not only time preferences literally but also storage costs, depreciation, and obsolescence of the stored products as well, yet on the other hand that most inventories are kept less than a year, it seems a reasonable compromise to presume that δ is not far off from 0.93365, the value which makes the duopolistic industry statistically neither pro- nor counter-cyclical when a varies just across the threshold \bar{a} . This helps us simplify the intuition, too. (This makes $\gamma \approx 0.28692a$, consistent with our original assumption $\gamma \geq 0.23a$.)

As a rough approximate benchmark, let the *status quo* mass of inventory investment be 0.5% of the economy’s GDP, and the de-trended fluctuations in the economy be in the order of 0.2% of the GDP. The macroeconomic fluctuations are assumed to be reflected on the demand intercept in every industry (whether oligopolistic or not). Considering the fact that the economic indicators are measured on the basis of the amount of transaction which, *ceteris paribus*, is generally proportional to the *square* of the demand intercept, the de-trended fluctuations in the demand intercept should be in the order of 0.1%.

When this skimpy 0.1% increment in the demand intercept occurs in our duopolistic industry, making the intercept a just cross the threshold \bar{a} , the amount of advance production suddenly grows from nil to $q^{AP}p^{AP}$, constituting the fraction

$$\frac{q^{AP} + q^{OS}}{q^{AP}} = \frac{2(3\sqrt{2} - 2)}{7} \approx 64\%$$

of the industry’s total production.

Let v be the weight of the duopolistic industry relative to the economy’s GDP. Suppose, for further simplicity, that the inventory fluctuations in all other industries are just proportional to the fluctuation of the total production, that is, 0.2% of the amount of inventories which is 0.5% of the production. In this economy, if aggregate inventory fluctuations represent 40% of the aggregate GDP fluctuation which is 0.2% of the GDP,

$$v \times 64\% + (1 - v) \times 0.5\% \times 0.2\% = 40\% \times 0.2\%$$

i.e., the weight v needs to be no more than 0.1234%, merely 1/810 of the economy. Note further that, if the inventory fluctuations in other industries are proportionately larger, the required weight of the duopolistic industry v becomes even smaller.

Our numerical example has demonstrated that the strategic incentives for inventory investment in an oligopolistic industry can explain the widely observed extreme volatility of macroeconomic inventories, even if the weight of such an oligopolistic industry is no more than a tiny — almost negligible — fraction of the economy.

4 Discussion on general oligopoly

In section 1 we have emphasised that our model, unlike much of preceding literature on capacity, can encompass Cournot, Bertrand, and possibly other forms of oligopoly. Thus far, we have taken a fairly thorough look at Cournot cases. Now we switch gears to non-Cournot oligopoly and discover what the major qualitative differences are. Throughout this section we shall not provide formal algebraic proofs, as we believe that graphical discussions suffice for the clarity of our qualitative results.

4.1 Bertrand market

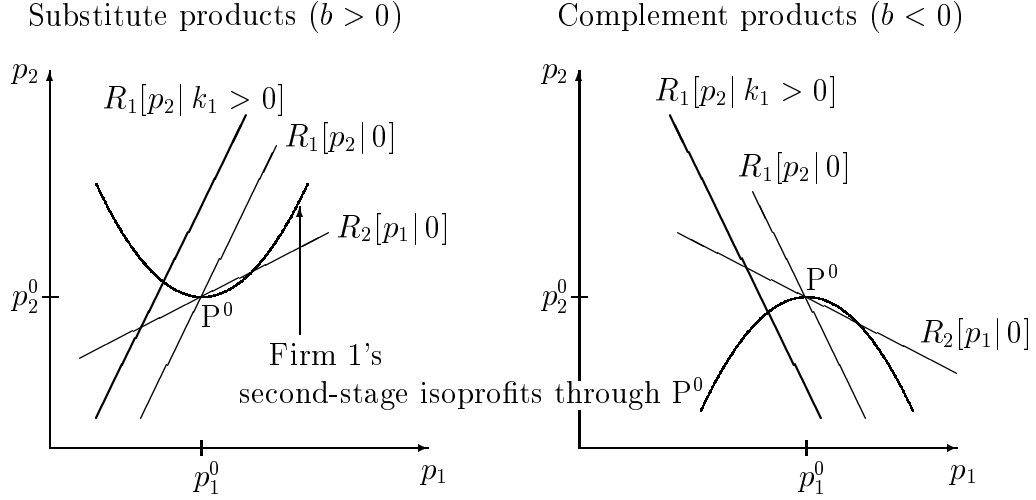
Revisit our benchmark model presented in section 2 except that we now replace quantity setting with price setting instead. The main *strategic* difference is the fact that, by advance production, a price setting firm's second-stage reaction curve would shift *inward*, because advance production sinks production costs in the first stage.¹⁰

Figure 3 presents a Bertrand analogue of figure 2 (see section 2). Note that figure 3 illustrates only the *direction of shifts* in firm 1's price-reaction curves made possible by means of advance production given $k_2 = 0$. The specific *shapes* of these reaction curves (e.g., the presence or absence of kinks or flat-spots) is immaterial to our qualitative observations hereinbelow. The left diagram of figure 3 represents the case where the two firms sell (possibly imperfect) substitute products (i.e., $b > 0$), whilst the right diagram represents the case where the two firms are complementary suppliers ($b < 0$). In either diagram, firm 1's advance production shifts its price-reaction curve from the thin locus to the thick locus which would induce the price equilibrium in the second

¹⁰We apply the same notation $R_i[\cdot]$ for both price- and quantity-reactions without further notice, as the danger of confusion is remote.

stage to drift in the direction of previously lower-profit side of the *status quo* isoprofit (the isoprofit with $k_1 = k_2 = 0$ passing through P^0).¹¹

Figure 3 : Firm 1's disincentives against advance production.



This clearly implies that an analogous intuition to that in section 2 can no longer apply in this Bertrand duopoly.

4.2 Price versus quantity competition

It is an insightful thought experiment to hypothesise a duopoly market where one firm sets a price whilst the other sets a quantity. Although it is not always easy to find a real industry or market where such hybrid competition actually takes place, it is extremely useful in shedding light on *each* oligopolist's strategic incentives for/against advance production.¹²

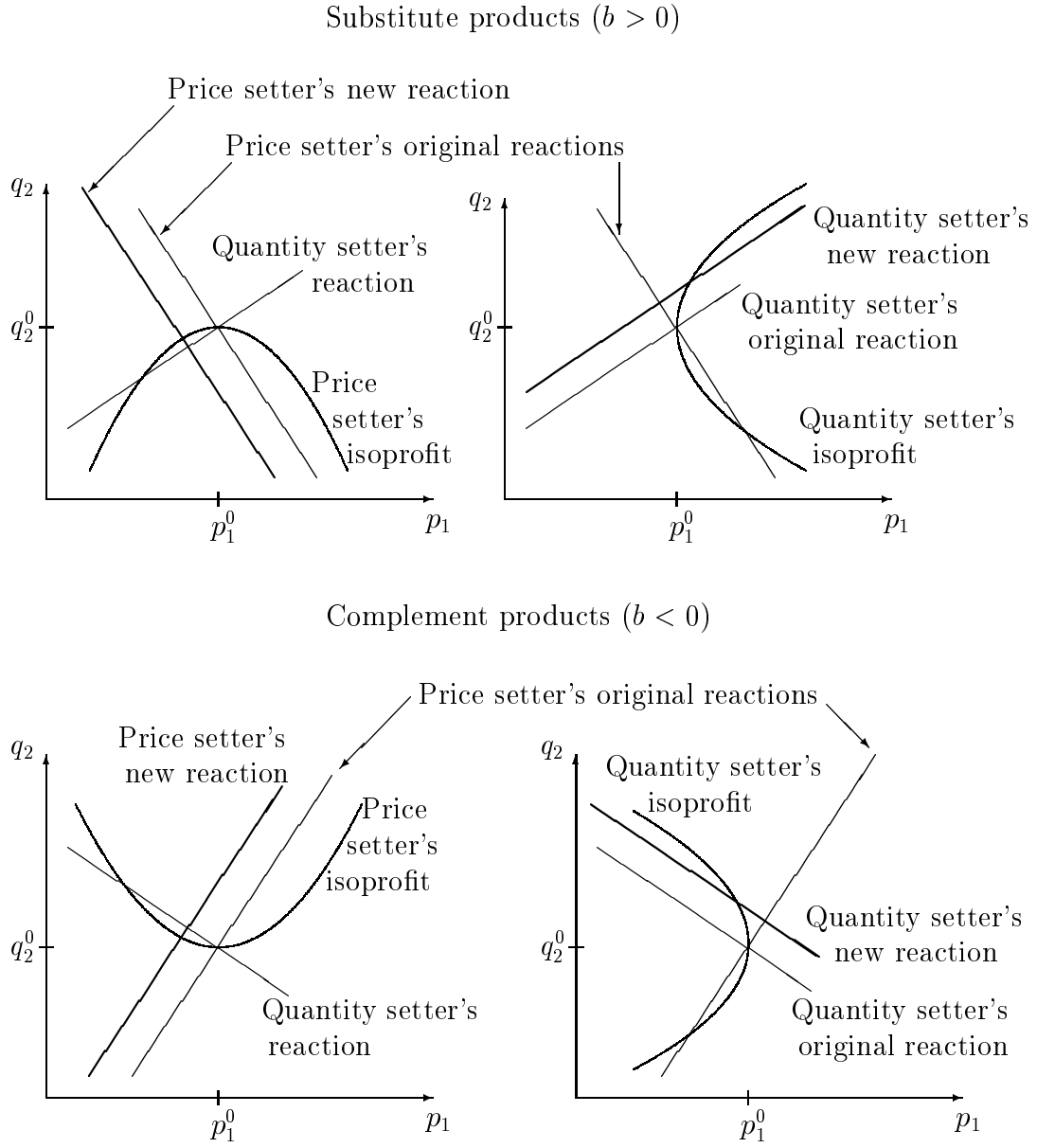
As we have already seen, by means of advance production the price setter can only shift its second-stage reaction curve *inward*, whereas the quantity setter can only

¹¹As in figure 2 (see footnote 5 in section 2) the exact position and shape of the isoprofit may depend upon k_1 . However, to our argument here, the only relevant isoprofit is the one with $k_1 = 0$.

¹²Game-theoretic attention has increasingly been drawn to this form of competition or, more precisely, to the qualitative comparison between this and other forms of competition such as Bertrand and Cournot. See Singh and Vives (1984), Boyer and Moreaux (1987), Lambertini (1997), Albæk and Lambertini (1998), *inter alia*.

shift its reaction curve *outward*. Hence figure 4, an analogue of figures 2 and 3. In either diagram therein, it is only the price setter not the quantity setter whose advance production can possibly drift the resulting second-stage equilibrium in the direction of the formerly higher-profit side (the side to which the original isoprofit is concave).

Figure 4 : Price setter's incentives for and quantity setter's disincentives against advance production



4.3 General proposition

All the foregoing observations imply that a firm can profitably invest in advance production *only when there is a quantity-setting opponent*, whether the firms' own choice variable is a price or a quantity, and whether the firms produce complements or substitutes.

Somewhat intriguingly, the key to determine firms' advance production incentives is not the super-/sub-modularity of the marketing game, but the strategic variable *per se*: whether price-setting or quantity-setting. The following summarises all the intuition we have developed in sections 2, 4.1 and 4.2. Advance production by a firm makes the firm strategically more prone to expand its sales quantity, which :

- reduces the *residual demand* for an opponent firm, who would therefore respond by cutting its price or quantity, if these two firms are substitute suppliers ;
- enhances the *residual demand* for an opponent firm, who would therefore respond by raising its price or quantity, if they are complement suppliers.

Hence, a price-setting opponent's reaction is always unequivocally unfavourable to the advance-producing firm, whilst a quantity-setting opponent's reaction can be a favourable one. Clearly, this intuition is not restricted to duopoly but can easily be extended to general oligopoly. In general, in order for a firm to undertake advance production profitably, there needs to be *at least one quantity-setting opponent* irrespective of complementarity or substitution between the firms' products.

5 Concluding remarks

In this paper, we have shown that the strategic advantage of advance production in oligopolistic competition may outweigh its intrinsic cost disadvantage, even without the prospect of Stackelberg leadership. Between Cournot oligopolists, advance production is strategically substitutional in that, under a well-behaved demand structure, advance production can be chosen only by some, not all, of the oligopolists in a pure strategy subgame perfect equilibrium whether firms are *a priori* symmetric or asymmetric. For

instance, in the case of duopoly, no more than one of the duopolists can profitably elect to invest in advance production, which entitles the firm to a competitive advantage in the Cournot-Nash equilibrium of the subsequent marketing stage.

In Bertrand competition, on the other hand, advance production can *never* be strategically advantageous so long as it is more costly than on-spot production. In general, for a firm to have any strategic incentives for costly advance production, there needs to be at least one *quantity setting competitor*.

These results do not depend upon whether firms' products are substitutes or complements perceived from the demand side. As an auxiliary remark, it is also noteworthy that advance production, whenever undertaken endogenously, contributes positively to welfare, the intuition similar to Stackelberg equilibria being welfare superior to simultaneous-move Cournot-Nash equilibria.

In more macroeconomic terms, this paper sheds light on firms' preferences between just-on-time production and investing in inventory. The presence of Cournot competition enhances oligopolists' strategic incentives in favour of building up inventory prior to the actual Cournot competition in the market, whilst Bertrand competition does not have similar implications.

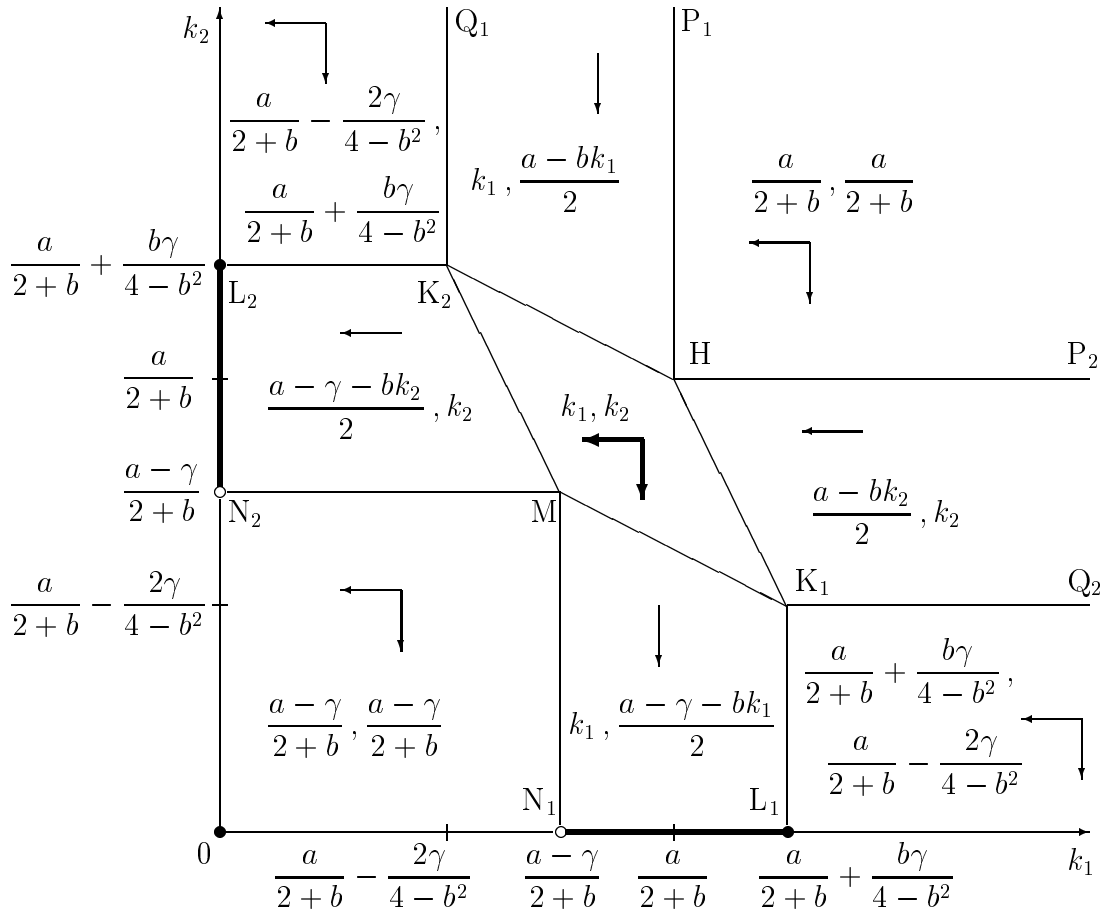
6 Appendix

The reaction function when $b \in (0, 1]$ is

$$R_i[q_j|k_i] = \begin{cases} \frac{a - bq_j}{2} & \text{if } \frac{a - 2k_i}{b} < q_j \leq \frac{a}{b}, \\ k_i & \text{if } \frac{a - \gamma - 2k_i}{b} \leq q_j \leq \frac{a - 2k_i}{b}, \\ \frac{a - \gamma - bq_j}{2} & \text{if } 0 \leq q_j < \frac{a - \gamma - 2k_i}{b}. \end{cases}$$

Based upon this reaction function, given k_1 and k_2 the equilibrium quantities q_1, q_2 in the second stage are summarised in figure 5. First and second coordinates indicate equilibrium q_1 and q_2 , respectively.

Figure 5 : Equilibrium quantities q_1, q_2 given k_1, k_2 when $b \in (0, 1]$.



Clearly, q_1 stays unaffected by k_1 to the right of the boundary $P_1HK_1L_1$, and to the left of the boundary $Q_1K_2MN_1$. Recalling our key assumption that advance production is strictly more costly than on-spot production, within these regions firm 1 has a strict incentive to minimise k_1 , indicated by thin left arrows in figure 5. Symmetrically, firm

2 has a similar incentives above $P_2HK_2L_2$ as well as below $Q_2K_1MN_2$, indicated by thin down arrows.

Furthermore, over the parallelogram HK_1MK_2 (borders included), based upon the quantity profile $q_1 = k_1$, $q_2 = k_2$, the discounted nett profits will be

$$\pi_i = \delta(a - k_i - bk_j)k_i - \gamma k_i \quad \{i, j\} = \{1, 2\},$$

whereby

$$\frac{\partial \pi_i}{\partial k_i} = \delta(a - 2k_i - bk_j) - \gamma \leq 0$$

over the entirety of the parallelogram insofar as $\delta < 1$. This leads to each firm's incentive to reduce k_i (thick down- and left-arrows in figure 5).

This confines the possible ranges of equilibrium advance production $\{k_1, k_2\}$ to the point of origin ($k_1 = k_2 = 0$) and the two symmetrical line segments L_1N_1 and L_2N_2 (highlighted by thick lines and filled dots in figure 5). By symmetry, we henceforth focus on the segment L_1N_1 and the point of origin.

- On the segment L_1N_1 , firm 1's discounted profit

$$\pi_1 = \delta k_1 \left(a - k_1 - b \cdot \frac{a - \gamma - bk_1}{2} \right) - \gamma k_1$$

is maximised either at the interior point $k_1 = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}$ if

$$\frac{a - \gamma}{2 + b} < \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta} < \frac{a}{2 + b} + \frac{b\gamma}{4 - b^2}, \quad (6)$$

or the end point $k_1 = \frac{a}{2 + b} + \frac{b\gamma}{4 - b^2}$ (L_1 in figure 5) if

$$\frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta} \geq \frac{a}{2 + b} + \frac{b\gamma}{4 - b^2}. \quad (7)$$

In these two cases, firm 1's maximised profits are respectively

$$\pi_1 \Big|_{k_1 = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}, k_2 = 0} = \frac{((2-b)\delta a - (2-b\delta)\gamma)^2}{8(2-b^2)\delta} \quad (8)$$

and

$$\pi_1 \Big|_{k_1 = \frac{a}{2+b} + \frac{b\gamma}{4-b^2}, k_2 = 0} = \delta \left(\frac{a}{2+b} + \frac{b\gamma}{4-b^2} \right)^2 - \gamma \left(\frac{a}{2+b} + \frac{b\gamma}{4-b^2} \right). \quad (9)$$

Otherwise, there is no maximum π_1 on the segment L_1N_1 if

$$\frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta} \leq \frac{a-\gamma}{2+b} \quad (10)$$

as the segment is *open* at its lower end N_1 . In this case, the profile $k_1 = k_2 = 0$ becomes an unambiguous equilibrium.

- If $k_1 = k_2 = 0$, then

$$\pi_1 \Big|_{k_1 = k_2 = 0} = \delta \left(\frac{a-\gamma}{2+b} \right)^2. \quad (11)$$

Hence no advance production $k_1 = k_2 = 0$ is a pure-strategy subgame perfect equilibrium if either

$$(10), \quad \text{or} \quad (6) \text{ and } (8) \leq (11), \quad \text{or} \quad (7) \text{ and } (9) \leq (11).$$

The former two correspond to (1) whilst the last one corresponds to (2).

Otherwise, if either

$$(6) \text{ and } (8) \geq (11), \quad \text{or} \quad (7) \text{ and } (9) \geq (11),$$

then there is a possibility that a pure-strategy subgame perfect equilibrium may lie on the segment L_1N_1 (and by symmetry, on the segment L_2N_2 as well). From figure 5 it is clear that firm 2's only possible deviation incentive from the segment L_1N_1 would be to deviate to the region $P_2HK_1Q_2$ (where there is no down-arrow). In this region, firm 2's discounted profit $\pi_2 = \left(\frac{(2-b)a - (2-b^2)k_2}{2} \delta - \gamma \right) k_2$ is maximised either at

$$\pi_2 \Big|_{k_2 = \frac{(2-b)\delta a - 2\gamma}{2(2-b^2)\delta}} = \frac{((2-b)\delta a - 2\gamma)^2}{8(2-b^2)\delta} \quad (12)$$

if

$$\frac{a-2k_1}{b} < \frac{(2-b)\delta a - 2\gamma}{2(2-b^2)\delta} < \frac{a}{2+b}, \quad (13)$$

or at

$$\pi_2 \Big|_{k_2 = \frac{a}{2+b}} = \delta \left(\frac{a}{2+b} \right)^2 - \frac{\gamma a}{2+b} \quad (14)$$

if

$$\frac{(2-b)\delta a - 2\gamma}{2(2-b^2)\delta} \geq \frac{a}{2+b}. \quad (15)$$

Namely, if firm 2 ever has any deviation incentive, it should deviate either to (12) or to (14). This implies the following.

- When (6) and (8) \geq (11) and hence $k_1 = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}$, if firm 2 stays on the segment L_1N_1 by setting $k_2 = 0$, the nett profit is

$$\pi_2 \left|_{k_1 = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}, k_2 = 0} = \frac{1}{\delta} \left(\frac{(4-2b-b^2)\delta a + (2b-(4-b^2)\delta)\gamma}{4(2-b^2)} \right)^2. \quad (16)$$

On the other hand, (15) cannot hold when (6), hence the only potential deviation in this case is to (12), which is *possible if and only if* (13) holds, which occurs if and only if

$$\frac{b^2 a}{2(2+b)\gamma} < \frac{1}{\delta} < \frac{b^2 a + 2b\gamma}{2(2+b)\gamma} \quad (17)$$

and, the deviation is *profitable if and only if* (12) is (strictly) higher than (16), which occurs if and only if

$$\frac{1}{\delta} < \frac{((2-b)\sqrt{2(2-b^2)} - 4 + 2b + b^2)a + (4-b^2)\gamma}{2(\sqrt{2(2-b^2)} + b)\gamma}. \quad (18)$$

- When (7) and (9) \geq (11) and hence $k_1 = \frac{a}{2+b} + \frac{b\gamma}{4-b^2}$, by setting $k_2 = 0$ and staying on the segment L_1N_1 , firm 2 can earn the nett profit

$$\pi_2 \left|_{k_1 = \frac{a}{2+b} + \frac{b\gamma}{4-b^2}, k_2 = 0} = \delta \left(\frac{(4-2b-b^2)a - 4\gamma}{2(4-b^2)} \right)^2. \quad (19)$$

Deviation to (12) is *possible if and only if* (13) holds, that is if and only if

$$\frac{b^2 a}{2(2+b)\gamma} < \frac{1}{\delta} < \frac{b^2(2-b)a + 4(2-b^2)\gamma}{2(4-b^2)\gamma} \quad (20)$$

and the deviation is *profitable if and only if* (12) $>$ (19), that is

$$\frac{1}{\delta} < \frac{(2-b)a}{2\gamma} + \frac{(2-b)\sqrt{2(2-b^2)}((-4+2b+b^2)a + 4\gamma)}{2(4-b^2)}. \quad (21)$$

Deviation to (14), on the other hand, is relevant if and only if (15) holds, that is

$$\frac{1}{\delta} \leq \frac{b^2 a}{2(2+b)\gamma} \quad (22)$$

and the deviation is *profitable if and only if* (14) $>$ (19), that is

$$\frac{1}{\delta} < \frac{4}{4-b^2} \left(1 - \frac{1}{2-b} \cdot \frac{\gamma}{a} \right). \quad (23)$$

Hence there is a pure-strategy subgame perfect equilibrium $k_i = \frac{\delta(2-b)a - (2-b\delta)\gamma}{(4-2b^2)\delta}$, $k_j = 0$ if

$$(6) \text{ and } (8) \geq (11) \text{ but not } ((17) \text{ and } (18)),$$

which corresponds to (3). Similarly, a pure-strategy subgame perfect equilibrium $k_i = \frac{a}{2+b} + \frac{b\gamma}{4-b^2}$, $k_j = 0$ exists if

$$(7) \text{ and } (9) \geq (11) \text{ but neither } ((20) \text{ and } (21)) \text{ nor } ((22) \text{ and } (23)),$$

which corresponds to (4).

Finally, no pure-strategy subgame perfect equilibrium exists if either

$$(6) \text{ and } (8) \geq (11) \text{ and } (17) \text{ and } (18)$$

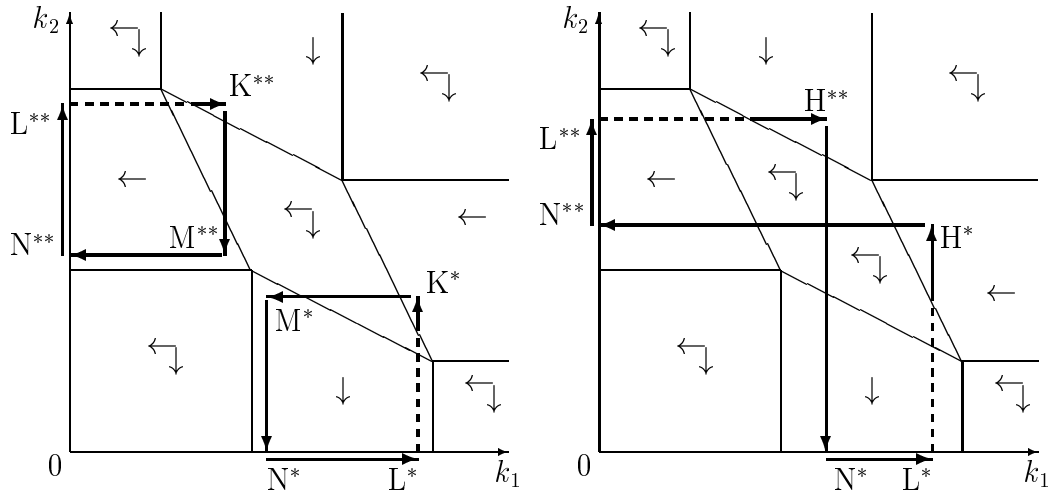
or

$$(7) \text{ and } (9) \geq (11) \text{ and either } ((20) \text{ and } (21)) \text{ or } ((22) \text{ and } (23))$$

which altogether corresponds to (5).

The following is a graphic intuition as to how the non-existence of pure strategy equilibria can occur. The two diagrams in figure 6 are a simplified version of figure 5.

Figure 6 : Cases where no pure equilibria exists.



In the left diagram, firm 1's best reaction against firm 2's $k_2 = 0$ is L^* , against which firm 2's best counterreaction is K^* . Note that, when either $\delta \uparrow 1$ or $\frac{\gamma}{a} \downarrow 0$ (which

occurs when $a \uparrow \infty$) all the down- and left-arrows in figure 6 would disappear. This means that, when δ and/or a is sufficiently large, the down- and left-ward gravity forces are sufficiently weakened, making it possible that firm 2 may prefer K^* to L^* . Clearly from the diagram, from firm 2's profit maximising viewpoint K^* is an interior local maximum whilst L^* is the corner local maximum given k_1 , hence preference for K^* over L^* occurs when, and only when, the cumulative downward gravity force over the dashed line segment above L^* is sufficiently small.

Then, against K^* , firm 1's incentive reduces k_1 down to M^* , against which firm 2 counterresponds by setting back to $k_2 = 0$, thereby resulting in a limit cycle. If an analogy holds also starting from $k_1 = 0$ instead, then there cannot exist any pure strategy equilibrium.

In the right diagram, the mechanism creating a limit cycle is similar to the previous one except that firm 2's counterreaction against L^* overshoots up to H^* , entailing firm 1's incentive toward $k_1 = 0$. In this case, there is one unified limit cycle in lieu of two disjoint cycles previously.

The reaction function when $b \in (-1, 0)$ is

$$R_i[q_j|k_i] = \begin{cases} \frac{a - bq_j}{2} & \text{if } 0 \leq q_j < \frac{2k_i - a}{-b}, \\ k_i & \text{if } \frac{2k_i - a}{-b} \leq q_j \leq \frac{2k_i + \gamma - a}{-b}, \\ \frac{a - \gamma - bq_j}{2} & \text{if } q_j > \frac{2k_i + \gamma - a}{-b}, \end{cases}$$

which entails figure 7 in lieu of figure 5. All the algebraic operations ensuing figure 5 can be directly applied to figure 7 as well, except :

- if $(7) \text{ and } (9) \geq (11)$ then L_1 must be an equilibrium ;
- if $(6) \text{ and } (8) \geq (11)$ then firm 2's only possible deviation incentive from the segment L_1N_1 would be to deviate to the trapezoid $L_2K_2MN_2$ (where there is no down-arrow). However, in this trapezoid, firm 2's discounted profit

$$\pi_2 = \left(\frac{((2-b)a + b\gamma - (2-b^2)k_2)}{2} \delta - \gamma \right) k_2 \text{ has a local maximum if and only if}$$

$$\frac{((2-b)a + b\gamma)\delta - 2\gamma}{2(2-b^2)\delta} > \frac{2k_1 + \gamma - a}{-b}. \quad (24)$$

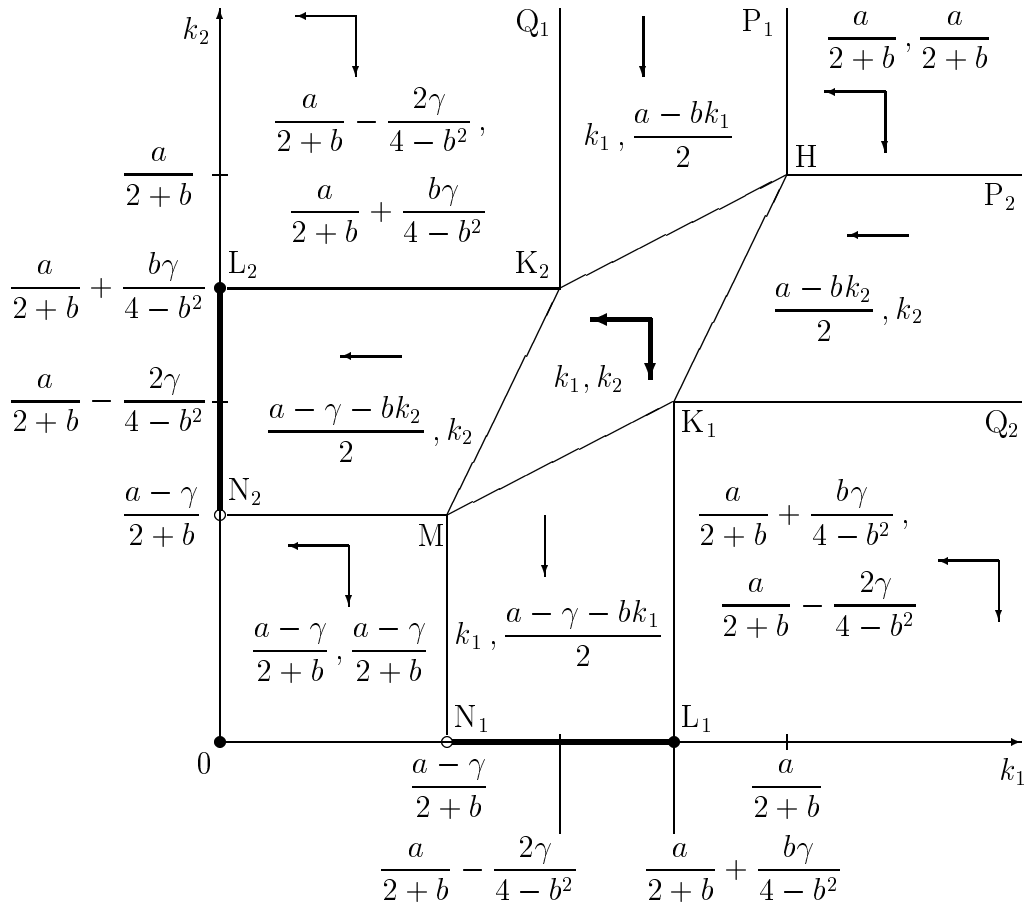
In (6), the optimal k_1 is $k_1 = \frac{((2-b)a + b\gamma)\delta - 2\gamma}{2(2-b^2)\delta}$, which makes inequality (24) inadmissible.

Hence, when $b \in (-1, 0)$, an equilibrium lies on the segment L_1N_1 (and an analogous equilibrium on L_2N_2) whenever either

$$(6) \text{ and } (8) \geq (11), \quad \text{or} \quad (7) \text{ and } (9) \geq (11).$$

Hence, the quadrichotomy [0] through [3] (section 2) stands valid even when $b \in (-1, 0)$ except, obviously, that regime [3] is now empty.

Figure 7 : Equilibrium quantities q_1, q_2 given k_1, k_2 when $b \in (-1, 0)$.



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